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$\mathcal{N} = 4$ super Yang Mills at finite density: the naked truth

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Abstract: We study $\mathcal{N} = 4$ super Yang Mills theory at finite $\mathrm{U}(1)_R$ charge density (and temperature) using the AdS/CFT Correspondence. The ten dimensional backgrounds around spinning D3 brane configurations split into two classes of solution. One class describe spinning black branes and have previously been extensively studied, and interpreted, in a thermodynamic context, as the deconfined high density phase of the dual field theory. The other class have naked singularities and in the supersymmetric limit are known to correspond to multi-centre solutions describing the field theory in the Coulomb phase. We provide evidence that the non-supersymmetric members of this class represent naked, spinning D-brane distributions describing the Coulomb branch at finite density. At a critical density a phase transition occurs to a spinning black brane representing the deconfined phase where the Higgs vevs have evaporated. We perform a free energy calculation to determine the phase diagram of the Coulomb branch at finite temperature and density.

Keywords: Black Holes in String Theory, Brane Dynamics in Gauge Theories, AdS-CFT and dS-CFT Correspondence.

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Contents

1. Introduction

In this letter we study $\mathcal{N} = 4$ super Yang Mills theory at finite $\mathrm{U}(1)_R$ charge density using the AdS/CFT Correspondence [1, 2, 3]. The Correspondence allows us to study this phenomena non-perturbatively by investigating the background to a stack of spinning D3 branes [4, 5, 6]. The spin induces non-zero components of the ten dimensional metric that after Kaluza Klein reduction on the S^5 correspond to a vacuum expectation value (vev) for a temporal $\mathrm{U}(1)_R$ gauge boson. In the field theory dual this field plays the role of a chemical potential putting the field theory at finite density.

A set of metrics describing spinning D3 branes have been obtained by oxidising five dimensional charged black hole solutions to ten dimensions [4]. These ten dimensional solutions break down into two classes. The first are rotating black branes and these have been extensively studied in the literature [4, 5, 6, 8]. In terms of the duality with the $\mathcal{N} = 4$ gauge theory they have been interpreted as the high density and high temperature deconfined phase. The phase structure for the $\mathcal{N} = 4$ theory at the origin of moduli space was mapped out in [6, 7]. The second class of solutions are not black branes but nakedly singular metrics. The supersymmetric limit of these solutions has been shown to correspond to disc distribution, multi-centre D3 brane solutions [5]. We will retell this story but using brane probing techniques to motivate moving to coordinates in which the duality is manifest, in the spirit of [9]–[12]. Our main interest though is in interpreting the non-supersymmetric members of this class. Since they share many properties with the rotating black branes it seems likely that they describe spinning disc distributions corresponding to the Coulomb branch of the gauge theory at finite density. We provide evidence that this is indeed the correct interpretation.

The non-supersymmetric naked solutions only exist upto some maximum density above which they develop a horizon and become the zero temperature black brane solutions. We interpret this transition, at which, as we will see, the role of the parameters of the model radically change, as the high density deconfinement transition of the Coulomb branch of the

JHEP07(2002)070

gauge theory above which the scalar vevs evaporate. We perform a free energy calculation comparing a spinning D3 distribution background with a compact time dimension, and a black brane geometry with the same temperature and density, to map out the phase diagram of the Coulomb branch. As one would expect the distribution size (ie the size of the scalar vev) controls the transition temperature and density.

As has been noted elsewhere [8] for the black brane geometries, these backgrounds are unstable if the spin or density is taken too large. We show this using a brane probe computation (note probes of some related configurations were performed in [13]). In fact the instability of the gauge theory at zero temperature and finite density is readily apparent perturbatively because the chemical potential destabilizes the scalar potential resulting in a run away vacuum [14]. The naive scalar instability in the theory is most clearly seen by observing that a probe D3 brane in pure $\mathrm{AdS}_5 \times S^5$ experiences no potential and thus there is no force to support rotational motion. Such a probe when given angular momentum moves off to infinity corresponding to a runaway scalar vev in the field theory. These geometries show that the instability remains non-perturbatively. Inspite of this instability, the geometries nevertheless let us see the physics of the finite density phase transition.

2. Finite density

The background we wish to study is the near horizon limit of a rotating D3 brane configuration obtained by Cvetic et al [4] from the lift of five dimensional charged black hole solutions

$$\begin{aligned} ds^2_{10} &= \sqrt{\tilde{\Delta}}\left[-\left(H_1H_2H_3\right)^{-2/3} f dt^2 + \left(H_1H_2H_3\right)^{1/3}\left(f^{-1}dr^2 + \frac{r^2}{L^2}dx^2_{//}\right)\right] + \\ &+ \frac{L^2}{\sqrt{\tilde{\Delta}}}\sum_{i=1}^{3} X_i^{-1}\left(d\mu_i^2 + \mu_i^2\left(d\phi_i + gA^i dt\right)^2\right) \end{aligned} \tag{2.1}$$

where the μ_i are three direction cosines and

$$f = -\frac{\mu}{r^2} + \frac{r^2}{L^2}H_1H_2H_3\,, \qquad \frac{1}{L^2} = \frac{1}{\sqrt{2m}\sinh\alpha}\,, \qquad \mu = \frac{2m}{L^2} \tag{2.2}$$

$$A^i = \frac{L(1-H_i^{-1})}{l_i \sinh\alpha}dt\,, \qquad H_i = 1 + \frac{l_i^2}{r^2} \tag{2.3}$$

$$\tilde{\Delta} = \left(H_1H_2H_3\right)^{1/3}\sum_i \frac{\mu_i^2}{H_i}\,, \qquad X_i = H_i^{-1}\left(H_1H_2H_3\right)^{1/3} \tag{2.4}$$

$$B_{(4)} = -\frac{r^4}{L^4}H_1H_2H_3\sum_i \frac{\mu_i^2}{H_i}dt\wedge d^3x + \frac{1}{\sinh\alpha}\left(\sum_i l_i\mu_i^2 d\phi_i\right)\wedge d^3x \tag{2.5}$$

At large r the solution asymptotes to $\mathrm{AdS}_5 \times S^5$ with the AdS radius L. We will keep L fixed in the following analysis. The solution then has four free parameters, the l_i and μ (or equivalently m or α). For the five dimensional black hole solutions the l_i are charges

JHEP07(2002)070

under three U(1) gauge symmetries and μ the temperature. We expect these parameters to lift to ten dimensions to be rotation parameters in the three distinct U(1) planes of the S^5 and the temperature. The temperature of the black hole is given by

$$2\pi T = \frac{1}{\sqrt{g_{rr}}}\frac{d}{dr}\sqrt{g_{tt}}\Big|_{r=r_h} \tag{2.6}$$

where r_h is the horizon radius which can be determined from where the function $f = 0$

$$r_H^4 H_1(r_H) H_2(r_H) H_3(r_H) = \mu L^2 \tag{2.7}$$

For our ten dimensional solutions we find

$$4\pi T = \frac{2}{L^2} r_H (H_1 H_2 H_3)^{1/2} \left(2 - \frac{1}{L\mu^{1/2}(H_1 H_2 H_3)^{1/2}} (l_1^2 H_2 H_3 + l_2^2 H_1 H_3 + l_3^2 H_1 H_2) \right) \tag{2.8}$$

We can solve these equations for a number of special cases to find the value of μ that corresponds to $T = 0$. For example when a single l_i is non-zero $T = 0$ corresponds to $\mu = 0$, for two equal non-zero l_i $T = 0$ corresponds to $\mu = l^4/L^2$ and when all three l_i are equal $T = 0$ corresponds to $\mu = 27l^4/4L^2$. For μ equal to these μ_c values and above the solutions have a singularity, orginating in the f function, which corresponds to the horizon of the black hole. As μ increases the black hole temperature increases. However, for $0 < \mu < \mu_c$ the solutions do not have a horizon but have a naked singularity at $r = 0$. We show this in the plots of figure 1 where g_{rr} is plotted against r at varying μ for the case when all three l_i are equal.

The black hole/brane solutions are closely related to those analysed in [6, 7] to describe the behaviour of $\mathcal{N} = 4$ super Yang Mills at finite temperature and density. In [6, 7] the three l_i were taken equal and the variant of the above metric where the Minkowski space slices of AdS are compactified on S^3 was considered.[1] The parameters l_i control the rotation speed of the black hole or the chemical potential in the field theory. The parameter μ controls the temperature of the black hole (or in the dual field theory) with $\mu = \mu_c$ corresponding to T=0. Following [6, 7] these black hole solutions should be interpreted as gravity duals of the field theory at the origin of moduli space across the full temperature and density plane (the origin is described by the usual AdS/CFT correspondence). The behaviour of a Wilson loop [19] in these backgrounds show that at finite chemical potential and temperature the theory lives in a distinct (deconfined) phase from the (confined) theory at the origin.

2.1 Spinning discs

What role then is there for the nakedly singular solutions when $\mu < \mu_c$? Traditional thinking would declare these backgrounds unphysical, however, recent developments have shown that naked singularities need not be pathological [15, 16] but may simply represent

[1]Placing the gauge theory on S^3 introduces an extra scale into the problem which enlarges the region of the thermodynamic temperature vs density plane where the confined phase survives [6]. On R^3 the phase transition to the deconfined phase occurs the moment that a temperature or chemical potenital is introduced.

JHEP07(2002)070

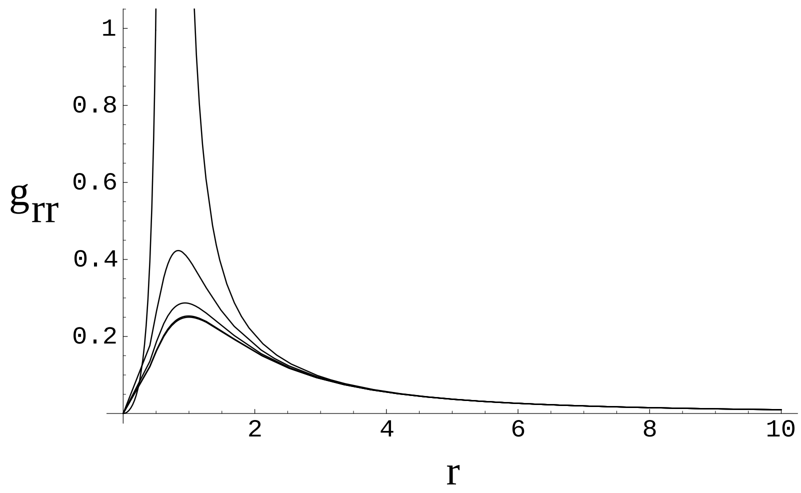

Figure 1: The metric component g_{rr} plotted as a function of radial position for varying μ up to $\mu = \mu_c$ at fixed $l_1 = l_2 = l_3 = l$ showing the development of a horizon.

JHEP07(2002)070

the presence of extended objects, such as D-branes, in the space. The most straight forward examples of such a background are the multi-centre solutions [5, 17, 18] describing a distribution of D3 branes which provide a dual description of the Coulomb branch of the $\mathcal{N} = 4$ gauge theory. In fact it has already been shown in [5] that the, supersymmetric, $\mu \to 0$ limit of precisely the singular backgrounds we consider here are multi-centre solutions. We begin with that analysis and will then consider what happens as μ is switched on.

One must be careful in taking the $\mu \to 0$ limit to remember to keep L fixed which also requires $\alpha \to \infty$. The background becomes

$$\begin{aligned} ds_{10}^2 &= \sqrt{\tilde{\Delta}} \left[(H_1 H_2 H_3)^{1/3} \frac{r^2}{L^2} \left(-dt^2 + dx_{//}^2 \right) + \frac{L^2 (H_1 H_2 H_3)^{-2/3}}{r^2} dr^2 \right] + \\ &\quad + \frac{L^2}{\sqrt{\tilde{\Delta}}} \sum_{i=1}^{3} X_i^{-1} \left(d\mu_i^2 + \mu_i^2 \, (d\phi_i)^2 \right) \qquad (2.9) \\ B_4 &= -\frac{r^4}{L^4} H_1 H_2 H_3 \sum_i \frac{\mu_i^2}{H_i} dt \wedge dx^3 \qquad (2.10) \end{aligned}$$

Note that the one form potential vanishes in this limit leaving a non-rotating solution. The difficulty with interpreting backgrounds as duals of gauge theory is though the familiar problem of finding the coordinates appropriate to the duality. Brane probing has proven itself to be an especially useful tool in this respect since it converts the background to the abelian gauge theory on the world volume of the probe where we can use field theory intuition to find the correct coordinates [9]–[12]. Thus we place a slow moving D3 brane in the background through the Born Infeld action

$$S = -\frac{\tau_3}{g_s} \int d^4\xi \sqrt{-\det g_{ab}} - \mu_3 \int B_4 \qquad (2.11)$$

where $\tau_3 = \mu_3 g_s^{-1}$ and g_{ab} is the pull back of the background to the world sheet. We find the action

$$L = \frac{1}{2}\left(\sum_i \frac{\mu_i^2}{H_i^2}\dot{r}^2 + \sum_i \frac{1}{2} r^2 H_i \left(\dot{\mu}_i^2 + \mu_i^2 \dot{\phi}_i^2\right)\right) \tag{2.12}$$

There is no potential obstructing motion of the probe in the six dimensional transverse space giving a strong hint that the theory is indeed the pure $\mathcal{N} = 4$ theory. In the coordinates appropriate to the duality we expect a canonical kinetic term for the six scalar fields on the probe suggesting we try the new coordinates

$$w^2 \tilde{\mu}_i^2 = (r^2 + l_i^2)\mu_i^2 \tag{2.13}$$

which render the $\dot{\phi}^2$ terms canonical. It follows that

$$w^2 = \sum_i (r^2 + l_i^2)\mu_i^2 \tag{2.14}$$

These are the coordinates identified in [5] that convert the metric to the familiar form of a multi-centre solution. They transform the probe action so that it has a flat metric on moduli space and leave the spacetime background in the form

$$ds_{10}^2 = H_D^{-1/2} dx_{//}^2 + H_D^{1/2} dw^2 \tag{2.15}$$

with

$$B_4 = -H_D^{-1} dt \wedge dx^3 \tag{2.16}$$

We may find the form of H_D from the g_{xx} component of the metric using the coordinate transformation in (2.13). For example, for a single l_i switched on we find

$$H_D^{-1} = \frac{1}{L^4}(w^2 - l^2\mu_1^2)^2\left(\mu_1^2 + H\mu_2^2 + H\mu_3^2\right), \qquad H = 1 + \frac{l^2}{w^2 - l^2\mu_1^2} \tag{2.17}$$

where

$$\mu_1^2 = \frac{w^2}{w^2 + l^2(1-\mu_1^2)}\tilde{\mu}_1^2, \qquad \mu_{2/3}^2 = \frac{w^2}{w^2 - l^2\mu_1^2}\tilde{\mu}_{2/3}^2 \tag{2.18}$$

and thus

$$\mu_1^2 = \frac{(w^2 + l^2) \pm \sqrt{(w^2 + l^2)^2 - 4l^2 w^2 \tilde{\mu}_1^2}}{2l^2} \tag{2.19}$$

The result is unenlightening, except that if we look in the ϕ_1 plane at $w = l$ by setting $\tilde{\mu}_1 = 1$, $\tilde{\mu}_{2/3} = 0$ which corresponds to $\mu_1 = 1$, $\mu_{2/3} = 0$ at $w = l$ and we find $H_D = 0$. The metric in this case is singular at $w = l$, or in the original coordinates $r = 0$. The singularity corresponds to the position of the D3 brane distribution responsible for the background - it is a disc in the ϕ_1 plane at $w = l$.

JHEP07(2002)070

Similar manipulations for the case with two equal l_i give

$$H_D^{-1} = \frac{1}{L^4}\left(w^2 - l^2(\mu_1^2+\mu_2^2)\right)^2\left(\mu_1^2+\mu_2^2+H\mu_3^2\right), \qquad H = 1 + \frac{l^2}{w^2 - l^2(\mu_1^2+\mu_2^2)} \tag{2.20}$$

Again looking at $w = l$ and setting $\tilde{\mu}_3 = 0$ ($\mu_3 = 0$) so $\tilde{\mu_1} + \tilde{\mu_2} = 1$ ($\mu_1 + \mu_2 = 1$) we find singularities in the four dimensional space described by the ϕ_1 and ϕ_2 planes corresponding to a spherical D3 distribution in that space. The case with three equal l_i gives the much simpler result

$$H_D^{-1} = w^4/L^4 \tag{2.21}$$

Here the distribution is an S^5 at $w = l$ as can be deduced from the fact that the r coordinates only extend to $r = 0$ or $w = l$ or by following the deformation of one of the above singular distribution as l_3 is switched on. Note that the S^5 distribution does not show up as singularities in H_D because it is an SO(6) singlet and hence does not appear in the supergravity because it is not an operator in a short multiplet. The space is $\mathrm{AdS}_5 \times S^5$ truncated at $w = l$.

Now we have identified the physical coordinates for $\mu = 0$ we can consider turning μ back on for a fixed distribution (fixed l_i). Turning on μ introduces spin or finite density in the field theory as can be seen by looking at the metrics at large w ($\simeq r$) where they look like AdS with a gauge potential

$$A_i \simeq \frac{l_i \mu^{1/2}}{r^2} \tag{2.22}$$

In this limit we may treat the solution as a five dimensional solution and calculate the $\mathrm{U}(1)_R$ charge in the interior. We can thus deduce a charge density in the dual field theory associated with each of the three $\mathrm{U}(1)_R$ subgroups of $\mathrm{SU}(4)_R$ which are proportional to $l_i\mu^{1/2}$. It seems reasonable to conclude that we are observing a solution describing a spinning version of the disc distribution.

We should be careful to check for evidence that no other deformations of the theory have occured. Let us first address this issue in the middle example above where two of the l_i are switched on with equal values. As μ switches on note that the g_{xx} component of the metric is unchanged by the inclusion of μ. The singularity locus in this component remains at the same place. Also the four form $dt \wedge dw^3$ piece is unchanged showing that the number of D3 branes in the interior is unchanged. Finally we note that μ introduces no angular dependence in the ϕ_1 or ϕ_2 plane. The conclusion of these facts is that μ does not change the angularly constant in the ϕ_1 and ϕ_2 planes, distribution of D3 branes at $w = l$.

Thus the metrics with $\mu < \mu_c$ seem to naturally describe spinning versions of the multi-centre solution corresponding to the dual $\mathcal{N} = 4$ theory being on its coulomb branch with a chemical potential. In fact it is clear that these metrics must describe such configurations because they are the unique solutions of the field equations with the symmetries of these systems (it is particularly clear that a spinning S^5 distribution of D3 branes will share the symmetries of a spinning black hole). This sharing of symmetries between the black hole solutions and rotating D-brane distributions explains why the two sets of solutions are naturally intertwined.

JHEP07(2002)070

2.2 Finite density phase transition

It is interesting that we can not increase the chemical potential to infinity for a fixed distribution (fixed l_i) and maintain a rotating distribution form for the solution — at $\mu = \mu_c$ there is a transition to a black brane and we lose all information about the interior structure. In the field theory at this critical density apparently knowledge of the scalar vevs is lost. Note that there is a sharp change in the interpretation of the parameters of the solution. When the interior is naked the solution must provide information about the interior structure which it does through the parameters l_i and then μ plays the role of rotation speed. Above the critical μ there is a black brane and knowledge of the interior structure is lost and so l_i switch to describing the rotation and μ describes the newly available parameter, temperature.

In the field theory dual we must be seeing the *finite density transition of the coulomb branch* where the scalar potential is forced to favour zero vevs. When the chemical potential is much less than the scalar vevs the vevs will be unaffected whilst when the chemical potential is much larger the theory should look like the deconfined phase of the theory at the origin of moduli space. The scale of the transition should be set by the size of the vevs (l_i) and indeed we have seen $\mu_c \sim l^4$. Above the critical density the spacetime is a black hole, a phase that has been identified with the deconfined phase of the field theory, as we would expect for the phase when the scalar vevs evaporate.

If we begin with a black brane metric with $\mu = \mu_c$ $(T = 0)$ and want to decrease the chemical potential we now realize there are two possibilities in the field theory. If the theory has small or zero vev it will remain in the deconfined phase as we decrease the density, else, if the theory has a large vev, then as we decrease the density below that vev the system should undergo a transition to the Coulomb phase. It's now clear that the dual background elegantly offers us both of these choices! We can decrease the density in two ways - either we keep $\mu = \mu_c$ and decrease l in which case we retain a black brane configuration corresponding to the first case in the field theory, or we can keep l fixed and decrease μ in which case we obtain a spinning multi-centre solution describing the coulomb phase.

The solutions with the three l_i equal fit this story equally well except that there is no singularity to monitor the position of the D3 branes as μ is switched on. Again by considering deformations of other singular configurations it is clear that the interpretation is the same as that just given. The metrics with a single l_i switched on, however, do not show this behaviour. In fact as we saw above the condition for a T=0 black brane is precisely $\mu = 0$ where the solution becomes a supersymmetric non-rotating disc distribution. For some reason these metrics do not provide us with *any* description of the rotating zero temperature states. Presumably this is just a failure of the completeness of these solutions rather than anything more subtle and we would expect similar behaviour on that part of the coulomb branch if only we had the appropriate metrics.

We note that this transition from the Coulomb phase to the deconfined phase is also apparent in the similar solutions in which the Minkowski space slices of AdS are compactified [6, 7]. Recently Myers and Tafjord [20] have argued that the nakedly singular

JHEP07(2002)070

metrics in that case correspond to distributions of giant gravitons. Again though above some critical angular momentum the solutions shift to black hole solutions showing that at high enough density the giant gravitons are forced to evaporate leaving a deconfined phase.

2.3 Stability

Many authors [8, 13] have studied the stability of the black brane solutions within the class of geometries under discussion and concluded that they are unstable for large densities. This is to be expected [14] since if, at zero temperature, we introduce a chemical potential into the $\mathcal{N} = 4$ gauge theory at tree level via a vev for the temporal component of a spurious $\mathrm{U}(1)_R$ gauge field then there will be a contribution to the scalar potential since the scalars are in the 6 of $\mathrm{SU}(4)_R$.

$$\Delta L = |D^\mu \phi| \to A_0^2 |\phi|^2 \tag{2.23}$$

A negative mass term is introduced for the scalar which will destabilize the moduli space of the theory, giving rise to a runaway potential. The same phenomena is apparent if we try to introduce rotation for a D3 probe in AdS space. Since there is no potential in the transverse space (as we saw in (2.12) above), rotational motion can not be supported and the D3 brane will progress to the edge of the moduli space displaying the runaway scalar vev. This argument is of course naive because quantum effects could stabilize the potential. The backgrounds to spinning D3 branes discussed above though provide a complete dual description of the field theory with a chemical potential and we may determine their stability by finding the potential seen by a probe in their background. As an example the resulting probe potential in the case where the three l_i are set equal, and its expansion for small $A_0 = A_i(w = l) = \mu^{1/2}/l$ is

$$V = \frac{w^4}{L^4}\left(1 - \sqrt{1 - \frac{L^2 A_0^2 l^2}{w^4}}\right) \simeq \frac{1}{2L^2} A_0^2 l^2 + \frac{1}{8}\frac{A_0^4 l^4}{w^4} + \cdots \tag{2.24}$$

The probe is forced to infinity by the potential (we plot the full expression in figure 2). We deduce that the whole configuration is indeed unstable since any of the D3s in the distribution can be considered as the probe. Remarkably, these backgrounds have though allowed us to explore the finite density behaviour of the Coulomb branch of the theory ignoring this instability.

3. Thermodynamics of the Coulomb branch

We will now extend our analysis to include finite temperature and map out the phase diagram of a point on the Coulomb branch, in the spirit of the Hawking Page transition [21]. For ease of calculation we will study points on the Coulomb branch where the global $\mathrm{SU}(4)_R$ symmetry is preserved; these are distributions in which the D3 branes live on an S^5. We will therefore study the naked solutions above with all three l_i equal and fixed. These solutions exist upto μ_c and have been identified above with an S^5 distribution of D3 branes spinning equally in the three transverse planes. To study these solutions at non-zero temperature

JHEP07(2002)070

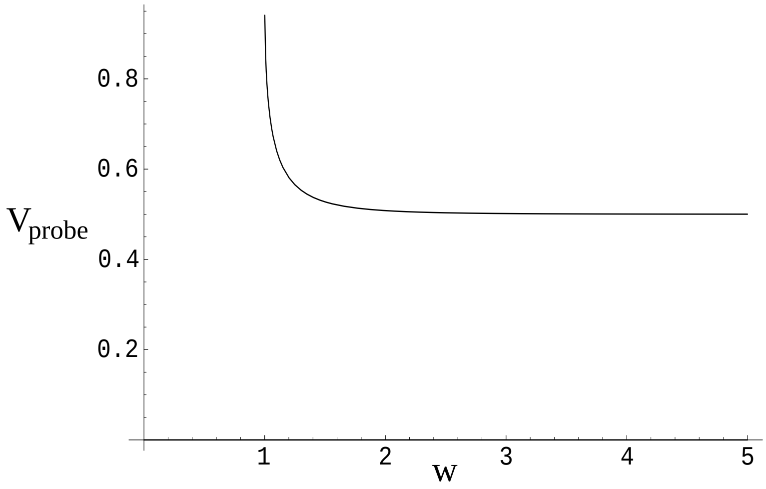

Figure 2: The form of the probe potential as a function of radial distance in the spinning D3 background.

we must compactify the time like direction with period $\beta = 1/T$. The chemical potential of these geometries is given by $\mu^{1/2}l$. The full set of geometries above also contain black brane solutions with the same temperature and chemical potential (above the chemical potential value $\mu_c^{1/2}l$ there are only black brane solutions so we study the parameter space below that point). To find which of these solutions is the energetically preferred solution we must calculate the free energy difference. The appropriate action is

$$I = -\frac{1}{2\kappa^2}\int d^{10}x\sqrt{-G}\left(R + \frac{1}{480}G^2_{(5)}\right) - \frac{1}{\kappa^2}\int d^9x\sqrt{-h_{(9)}}\overline{K} \tag{3.1}$$

The second integral is a surface term where

$$\overline{K} = G^{\mu\nu}K_{\mu\nu}\,, \qquad K_{\mu\nu} = \frac{1}{2\sqrt{G_{rr}}}\frac{\partial}{\partial r}G_{\mu\nu}\,, \qquad h_{(9)} = \det G_{\mu\nu}\,, \qquad \mu,\nu \neq r \tag{3.2}$$

As described in [21, 19], to allow comparison of the two spacetimes the period of the time integral of the naked solution, $\tilde{\beta}$, must be set to match the geometry of the hypersurface at large R in the two cases. To achieve this we require

$$\tilde{\beta} = \beta\frac{\sqrt{G_{tt}}}{\sqrt{\tilde{G}_{tt}}} \tag{3.3}$$

For the metrics under consideration calculation shows that the curvature, $R = 0$, leaving us with just the five-form and surface pieces. We use subscripts on the μ and l parameters to distinguish the naked and black brane cases; a 1 subscript denotes the black brane and a 2 the naked geometry. Direct computation gives the action difference

$$I = I_1 - I_2 = \frac{1}{\kappa^2}Vol(S_5)Vol(3)\beta\left(2l_2^4 - 2l_1^4 - \mu_2 + \mu_1 - 2r_h^4 - 4r_h^2l_1^2\right) \tag{3.4}$$

where $Vol(S_5)$ is the volume element associated with the angular integration which is common to the two geometries, $Vol(3)$ is the volume of the spatial part of the branes and r_h is the horizon radius of the black hole.

JHEP07(2002)070

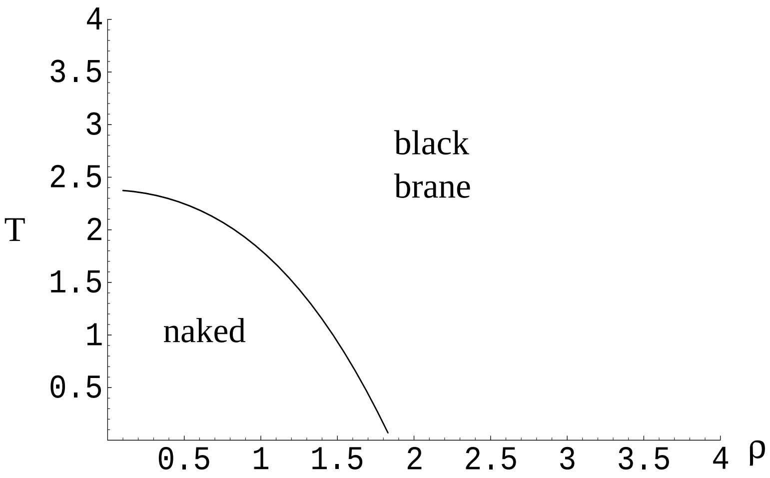

Figure 3: The temperature-density plane, showing the critical line inside which one has the naked solutions (Coulomb phase), and beyond it the black branes (deconfined phase)

The action calculation indeed reveals a phase transition between the two geomtries as a function of temperature and chemical potential. For low temperature and density the naked solution is preferred whilst for high temperature and density the black brane solution is preferred. The transition occurs essentially when the black brane radius becomes larger than the distribution size as expected, since these are the only two scales in the problem. We note that the transition on the zero temperature axis actually occurs a little below the value $\mu_c = 27/4l^4$ determined earlier. Thermodynamically there is no reason why this shouldn't be true - the phase diagram still matches expectations. It does though make the precise interpretation of μ_c more opaque. In the dual field theory at low temeprature and density the solution describes a point on the Coulomb branch with scalar vevs. At high temperature and density the theory transitions to a deconfined phase without scalar vevs. The transition occurs when the temperature or chemical potential is of order the scalar vevs. In figure 3 we plot the form of the phase diagram where it can be seen that the result of the supergravity calculation matches field theory expectations.

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JHEP07(2002)070

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